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Supersymmetric (non-)abelian bundles in the type-I and SO(32) heterotic string

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Abstract: We discuss perturbative four-dimensional compactifications of both the SO(32) heterotic and the type-I string on smooth Calabi-Yau manifolds endowed with general non-abelian and abelian bundles. We analyse the generalized Green-Schwarz mechanism for multiple anomalous U(1) factors and derive the generically non-universal one-loop threshold corrections to the gauge kinetic function as well as the one-loop corrected Fayet-Iliopoulos terms. The latter can be interpreted as a stringy one-loop correction to the Donaldson-Uhlenbeck-Yau condition. Applying S-duality, for the type-I string we obtain the perturbative Π-stability condition for non-abelian bundles on curved spaces. Some simple examples are given, and we qualitatively discuss some generic phenomenological aspects of this kind of string vacua. In particular, we point out that in principle an intermediate string scale scenario with TeV scale large extra dimensions might be possible for the heterotic string.

Keywords: Anomalies in Field and String Theories, Superstrings and Heterotic Strings, String Duality, Superstring Vacua.

Contents

1. Introduction

Over the past two decades, model building techniques have been developed in several corners of the M-theory moduli space. While early attempts focused on the heterotic $E_8 \times E_8$ string, the last ten years have seen considerable progress on the type-I side, both in the framework of intersecting D-branes (for a recent review see [1] and references therein) and its T-dual formulation of magnetised D-branes.

In view of the conjectured S-duality between the heterotic SO(32) and type-I string theories, the concrete constructions studied so far in both frameworks exhibit a couple of features which at first sight appeared to be not quite compatible [2, 3]:

- The common perception seems to be that on the type-I side one generically encounters multiple anomalous U(1) gauge symmetries, whose anomalies are cancelled by a generalised Green-Schwarz mechanism involving the Kähler-axion supermultiplets (also called internal axions in the remainder of this article) [2, 4]. For the heterotic string, by contrast, almost all previous models show at most one anomalous U(1), whose anomaly is cancelled by the universal axion complexifying the four-dimensional dilaton. This includes heterotic orbifold, free-fermion and Gepner model constructions.

- For the known heterotic examples the Fayet-Iliopoulos (FI) term vanishes identically at string tree level, but often receives a non-vanishing contribution generated at one loop [5, 6]. The latter can only be cancelled by giving vacuum expectation values (VEVs) to scalar fields charged under the U(1) in question. On the type-I side, the generic string tree level Fayet-Iliopoulos term is non-vanishing but contains terms at leading and next-to-leading order in α' [7]. Being also moduli dependent, it vanishes for certain choices of the moduli without invoking any charged scalars.

- In the heterotic constructions the gauge threshold corrections for the non-abelian gauge factors are universal [8], whereas for the type-I string each stack of D-branes comes with its own independent gauge coupling [9].

- The known heterotic string examples possess no adjoint scalars, which in turn are a common feature of the type-I string examples.

- Last but not least, one has got used to the perception that on the heterotic side the string scale is always close to the Planck scale, whereas for D-brane models the string scale is in principle a free parameter and the large Planck scale is an effect of large extra (transversal) dimensions.

The point we would like to make is that these statements are not generically true for all four-dimensional heterotic string compactifications, but are valid only under the restricted assumptions under which they were derived. More concretely, in [10] we argued that the heterotic and type-I string vacua investigated so far were different from the very beginning so that one could not expect to get similar results. On the type-I side one was considering various D-branes equipped with abelian gauge bundles (magnetized branes), whereas for the heterotic string one was mostly invoking non-abelian gauge bundles on Calabi-Yau manifolds often at their orbifold limits.

Therefore, in order to elucidate the possible difference between type-I and heterotic models, one had better make sure that one indeed compares S-dual constructions. This demonstrates the importance of studying heterotic string compactifications on Calabi-Yau manifolds equipped also with abelian gauge bundles[1] as well as type-I strings with non-abelian vector bundles on the world-volume of the D9-branes. In order to avoid complications with orbifold singularities and fixed point resolutions, it is much more transparent to work on smooth Calabi-Yau spaces and think about issues of orbifolds later.

In [10], building upon earlier work [17], it was shown for the $E_8 \times E_8$ heterotic string that multiple anomalous U(1)-factors occur in the four dimensional compactifications if the compact six-dimensional manifold is endowed with non-trivial line bundles. In this scenario, also the gauge kinetic function for the abelian observable gauge group factors turns out to be non-universal and there is a moduli dependent non-vanishing tree-level and one-loop contribution to the Fayet-Iliopoulos terms. This one-loop correction to the supersymmetry condition for the bundles allows for non-trivial models even on a Calabi-Yau with only one Kähler modulus, freezing a combination of the dilaton and the Kähler modulus by supersymmetry.

The aim of this article is to continue along the lines of [10] and to investigate four-dimensional Calabi-Yau compactifications of the heterotic SO(32) string equipped with not only non-abelian but also abelian bundles. We elaborate on the general framework for constructing such models in the perturbative, i.e. small string coupling and large radii, regime. Special emphasis will be put on clarifying the general structure that arises in the sector of the abelian gauge symmetries. We will work out the Green-Schwarz mechanism and demonstrate how in general it invokes both the universal and the internal axions. We will also compute the tree-level and one-loop threshold corrected gauge kinetic functions and find them to be non-universal. Furthermore, we will derive the perturbatively exact Fayet-Iliopoulos terms, which again contain a tree-level and a one-loop contribution. By S-duality these are precisely related to the perturbative part of the Π-stability condition [18]

$$\xi \sim \mathrm{Im}\left(\int_{\mathcal{M}} \mathrm{tr}_{\mathrm{U}(N)} \left[e^{i\varphi}\, e^{J\,\mathrm{id}+i\mathcal{F}} \sqrt{\hat{A}(\mathcal{M})} \right] \right) = 0 \tag{1.1}$$

for non-abelian $\mathrm{U}(N)$ bundles on manifolds with non-vanishing curvature. To our knowledge the precise form of this expression has so far never been derived from first principles and we find it amusing though not unexpected that it is already implicit in the celebrated Green-Schwarz anomaly terms [19].

The paper is organised as follows: We start in section 2 by discussing the four-dimensional spectrum of heterotic SO(32) compactifications for general $\mathrm{U}(N)$ bundles and by computing the field theory anomalies. In section 3, the corresponding Green-Schwarz mechanism involving the dilaton-axion and Kähler-axion multiplets is discussed in very much detail. Section 4 contains the computation of the non-universal gauge kinetic function, and in section 5, the one-loop corrected supersymmetry condition is derived. Section 6 is devoted to a discussion of S-duality to the type-I string with magnetised D-branes including an analysis of the appropriate stability conditions. In section 7, we finally give two simple examples which are supposed to serve merely as appetizers for the rich structure behind the new model building perspectives. A more detailed study of realistic models remains to be endeavoured. In section 8 we briefly address some phenomenological issues for

[1] See [11 – 16] for earlier studies of models of this type.

this class of heterotic string compactifications, restricting ourselves mostly to a qualitative level. In particular we point out that for certain choices of the internal bundles it might be possible to lower the string scale down to the intermediate regime even for heterotic strings. Our conclusions are displayed in section 9. Some technical details for the computations of sections 2 and 3 are relegated to appendices A and B, respectively.

2. Breaking of SO(32) via unitary bundles

The aim of this section is to define a general class of compactifications of the SO(32) heterotic string involving direct sums of bundles with unitary structure groups. This includes direct sums of abelian bundles, i.e. line bundles, which have mainly been addressed in the S-dual models of magnetised D-branes respectively intersecting D-branes in the mirror symmetric setting.

Concretely, we consider decompositions of SO(32) into

$$\mathrm{SO}(2M) \times \prod_{x=1}^{K+L} \mathrm{U}(N_x) \tag{2.1}$$

with $M + \sum_{x=1}^{K+L} N_x = 16$. One can realise such a breaking of the original SO(32) by giving a background value to bundles on the internal manifold with structure group $G = \mathrm{U}(1)^K \times \prod_{m=K+1}^{K+L} \mathrm{U}(N_m)$, i.e. by turning on bundles of the form

$$W = \bigoplus_{m=K+1}^{K+L} V_m \oplus \bigoplus_{i=1}^{K} L_i\,. \tag{2.2}$$

Note that in the following the index i always runs over the range $\{1,\ldots,K\}$, the index m over the range $\{K+1,\ldots,K+L\}$ and the index x over the entire range $\{1,\ldots,K+L\}$. The resulting observable non-abelian gauge group is $H = \mathrm{SO}(2M) \times \prod_{j=1}^{K} \mathrm{SU}(N_j)$ and the anomaly free part of the $\mathrm{U}(1)^{K+L}$ factors also remains in the low energy gauge group.

The second choice is to include the $\mathrm{SO}(2M)$ factor in the bundle, i.e. to start with

$$W = V_0 \oplus \bigoplus_{m=K+1}^{K+L} V_m \oplus \bigoplus_{i=1}^{K} L_i \tag{2.3}$$

with $c_1(V_0) = c_3(V_0) = 0$ and the observable non-abelian part of the gauge group $H = \prod_{j=1}^{K} \mathrm{SU}(N_j)$.

The general model building constraints at tree and stringy one-loop level for $E_8 \times E_8$ have been discussed at length in [10]. Let us give the corresponding conditions for SO(32) here:

- In order to admit spinors, we need $c_1(W) \in H^2(\mathcal{M}, 2\mathbb{Z})$.
- At string tree level, the field strength of the vector bundle has to satisfy the zero-slope limit of the hermitean Yang-Mills equations, $F_{ab} = F_{\bar{a}\bar{b}} = 0$, $g^{a\bar{b}} F_{a\bar{b}} = 0$, constraining to μ-stable, holomorphic vector bundles which satisfy the integrability condition $\int_{\mathcal{M}} J \wedge J \wedge c_1(V) = 0$ for each constituent V of the total bundle.

- The non-holomorphic hermitean Yang-Mills equation is modified at one loop. As we will compute in section 5, for a $\mathrm{U}(N)$ bundle V the perturbatively exact integrability condition reads

$$\frac{1}{2}\int_{\mathcal{M}} J\wedge J\wedge c_1(V) - g_s^2\,\ell_s^4 \int_{\mathcal{M}} \Big(\mathrm{ch}_3(V) + \frac{1}{24}\, c_1(V)\, c_2(T)\Big) = 0\,, \tag{2.4}$$

where $g_s = e^{\phi_{10}}$ and $\ell_s = 2\pi\sqrt{\alpha'}$. An additional constraint arises from the requirement that the one-loop corrected $\mathrm{U}(1)$ gauge couplings are real,

$$\frac{N}{3!}\int_{\mathcal{M}} J\wedge J\wedge J - g_s^2\,\ell_s^4 \int_{\mathcal{M}} J\wedge \Big(\mathrm{ch}_2(V) + \frac{N}{24}\, c_2(T)\Big) > 0\,, \tag{2.5}$$

in the perturbative regime, i.e. for $g_s \ll 1$ and $r_i \gg 1$. Note that there will be additional stringy and α' non-perturbative corrections to (2.4) and (2.5).

- The Bianchi identity for the three-form $H = dB - \frac{\alpha'}{4}(\omega_Y - \omega_L)$ imposes the 'tadpole condition' $dH = \frac{\alpha'}{4}(\mathrm{tr}(R^2) - \mathrm{tr}(F^2))$, where the traces are taken in the fundamental representation of $\mathrm{SO}(1,9)$ and $\mathrm{SO}(32)$, respectively. For decompositions of the type (2.2), this condition reads

$$0 = c_2(T) + \sum_{i=1}^{K} N_i\, \mathrm{ch}_2(L_i) + \sum_{m=K+1}^{K+L} \mathrm{ch}_2(V_m) \tag{2.6}$$

in cohomology.

- The chiral spectrum can be determined from the Euler characteristics of the various bundles $\mathcal{W}$ occurring in the decomposition of $\mathrm{SO}(32)$,

$$\chi(\mathcal{M},\mathcal{W}) = \int_{\mathcal{M}} \Big[\mathrm{ch}_3(\mathcal{W}) + \frac{1}{12}\, c_2(T)\, c_1(\mathcal{W})\Big]\,. \tag{2.7}$$

In the remainder of this section, we present the generic chiral spectrum and compute the field theoretical four-dimensional anomalies. In section 3, we show that these anomalies are exactly cancelled by the dimensional reduction of the ten-dimensional kinetic and one-loop Green-Schwarz counter terms.

The above models can be studied for arbitrary N_x:[2] The adjoint representation of $\mathrm{SO}(32)$ decomposes as follows

$$\mathbf{496} \longrightarrow \left(\begin{array}{c} (\mathbf{Anti}_{\mathrm{SO}(2M)})_0 \\ \sum_{x=1}^{K+L} (\mathbf{Adj}_{\mathrm{SU}(N_x)})_0 + (K+L)\times(\mathbf{1})_0 \\ \sum_{x=1}^{K+L} (\mathbf{Anti}_{\mathrm{SU}(N_x)})_{2(x)} + \text{h.c.} \\ \sum_{x<y} \big[(\mathbf{N}_x, \mathbf{N}_y)_{1(x),1(y)} + (\mathbf{N}_x, \overline{\mathbf{N}}_y)_{1(x),-1(y)} + \text{h.c.}\big] \\ \sum_{x=1}^{K+L} (2M, \mathbf{N}_x)_{1(x)} + \text{h.c.} \end{array}\right). \tag{2.8}$$

The massless spectrum therefore takes the form given in table 1 for bundles of type (2.2). For bundles of the form (2.3), the states transforming under $\mathrm{SO}(2M)$ are computed from table 2.

reps.	$H = \prod_{j=1}^{K} \mathrm{U}(N_j) \times \mathrm{U}(1)^L \times \mathrm{SO}(2M)$
$(\mathbf{Adj}_{\mathrm{U}(N_j)})_{0(j)}$	$H^*(\mathcal{M}, \mathcal{O})$
$(\mathbf{1})_{0(m)}$	$H^*(\mathcal{M}, V_m \otimes V_m^*)$
$(\mathbf{Anti}_{\mathrm{SU}(N_j)})_{2(j)}$	$H^*(\mathcal{M}, L_j^2)$
$(\mathbf{1})_{2(m)}$	$H^*(\mathcal{M}, \bigwedge^2 V_m)$
$(\mathbf{N}_i, \mathbf{N}_j)_{1(i),1(j)}$	$H^*(\mathcal{M}, L_i \otimes L_j)$
$(\mathbf{N}_i, \overline{\mathbf{N}}_j)_{1(i),-1(j)}$	$H^*(\mathcal{M}, L_i \otimes L_j^{-1})$
$(\mathbf{N}_i)_{1(i),1(m)}$	$H^*(\mathcal{M}, V_m \otimes L_i)$
$(\mathbf{N}_i)_{1(i),-1(m)}$	$H^*(\mathcal{M}, V_m^* \otimes L_i)$
$(\mathbf{1})_{1(m),1(n)}$	$H^*(\mathcal{M}, V_m \otimes V_n)$
$(\mathbf{1})_{1(m),-1(n)}$	$H^*(\mathcal{M}, V_m \otimes V_n^*)$
$(\mathbf{Adj}_{\mathrm{SO}(2M)})$	$H^*(\mathcal{M}, \mathcal{O})$
$(2M, \mathbf{N}_j)_{1(j)}$	$H^*(\mathcal{M}, L_j)$
$(2M)_{1(m)}$	$H^*(\mathcal{M}, V_m)$

Table 1: Massless spectrum. $M \equiv 16 - \sum_{x=1}^{K+L} N_x$. The indices run over the range $i, j \in \{1, \dots, K\}$ and $m, n \in \{K+1, \dots, K+L\}$. The structure group is taken to be $G = \mathrm{U}(1)^K \times \prod_{m=K+1}^{K+L} \mathrm{U}(N_m)$.

reps.	$H = \prod_{j=1}^{K} \mathrm{U}(N_j) \times \mathrm{U}(1)^L$
$(\mathbf{1})$	$H^*(\mathcal{M}, \bigwedge^2 V_0)$
$(\mathbf{N}_j)_{1(j)}$	$H^*(\mathcal{M}, V_0 \otimes L_j)$
$(\mathbf{1})_{1(m)}$	$H^*(\mathcal{M}, V_0 \otimes V_m)$

Table 2: Massless spectrum for $G = \mathrm{U}(1)^K \times \prod_{m=K+1}^{K+L} \mathrm{U}(N_m) \times \mathrm{SO}(2M)$. These three lines replace the last three lines in table 1, all other cohomology classes are identical in both cases.

Note that this is the same massless spectrum as for the perturbative type-I string on a smooth Calabi-Yau space with B-type D9-branes. In particular, turning on abelian bundles for the SO(32) heterotic string also leads to $H^1(\mathcal{M}, \mathcal{O})$ massless chiral multiplets transforming in the *adjoint* representation of a $\mathrm{U}(N_j)$ observable gauge factor. In many cases there do not exist any non-trivial homological one-cycles but on the torus one has for instance $H^1(T^6, \mathcal{O}) = 3$. These complex adjoint scalars correspond to the continuous Wilson lines on $\mathcal{M}$. Analogously, turning on non-abelian bundles on the type-I D9-branes gives rise to $H^1(\mathcal{M}, (V_m \otimes V_m^*))$ moduli corresponding to the deformations of the $\mathrm{U}(N_m)$ bundle.

Now let us discuss the resulting anomalies. We will first consider the case with structure group $G = \mathrm{U}(1)^K \times \prod_{m=K+1}^{K+L} \mathrm{U}(N_m)$ and the resulting observable gauge group $H = \mathrm{SO}(2M) \times \prod_{j=1}^{K} \mathrm{SU}(N_j) \times \mathrm{U}(1)^{K+L}$. The anomalies are computed from the net number of chiral multiplets of the diverse representations $\mathcal{W}$ using (2.7). For the cubic non-abelian anomalies we obtain from

$$\mathcal{A}_{\mathrm{SU}(N_i)^3} \sim (N_i - 4)\chi(L_i^2) + \sum_{j \neq i} N_j \left(\chi(L_i \otimes L_j) + \chi(L_i \otimes L_j^{-1}) \right) +$$

[2] The decomposition of the adjoint of SO(32) is the same as in the case where in type-IIA orientifold theory all D6-branes first lie on top of the O6-planes and then stacks of N_x branes are rotated away from this position.

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$$+\sum_m \left(\chi(V_m \otimes L_i) + \chi(V_m^* \otimes L_i)\right) + 2M\chi(L_i)\,, \tag{2.9}$$

the expression in terms of Chern characters,

$$\mathcal{A}_{\mathrm{SU}(N_i)^3} \sim 2\int_{\mathcal{M}} c_1(L_i) \times \mathrm{Tad}\,, \tag{2.10}$$

with the tadpole condition

$$\mathrm{Tad} = c_2(T) + \sum_{j=1}^{K} N_j\, \mathrm{ch}_2(L_j) + \sum_{m=K+1}^{K+L} \mathrm{ch}_2(V_m) = 0 \tag{2.11}$$

in cohomology. Thus in contrast to the $E_8 \times E_8$ examples discussed in [10], the cubic non-abelian anomalies vanish only upon tadpole cancellation [20].

The explicit expressions for all mixed and cubic abelian anomalies in terms of Euler characteristics are given in appendix A. Here we only state the result in terms of the various Chern characters up to tadpole cancellation for the $\mathrm{U}(1)_i$ factors with $i \in \{1, \dots, K\}$

$$\begin{aligned}
\mathcal{A}_{\mathrm{U}(1)_i-\mathrm{SU}(N_j)^2} &\sim \int_{\mathcal{M}} N_i\, c_1(L_i) \left(\frac{1}{6}\, c_2(T) + 2\,\mathrm{ch}_2(L_j)\right) + 2\int_{\mathcal{M}} N_i\, \mathrm{ch}_3(L_i)\,,\\
\mathcal{A}_{\mathrm{U}(1)_i-G_{\mu\nu}^2} &\sim \frac{1}{2}\int_{\mathcal{M}} N_i\, c_1(L_i)\, c_2(T) + 24\int_{\mathcal{M}} N_i\, \mathrm{ch}_3(L_i)\,,\\
\mathcal{A}_{\mathrm{U}(1)_i-\mathrm{U}(1)_j^2} &\sim N_j \int_{\mathcal{M}} N_i\, c_1(L_i) \left(\frac{1}{6}\, c_2(T) + 2\mathrm{ch}_2(L_j)\right) + 2\, N_j \int_{\mathcal{M}} N_i\, \mathrm{ch}_3(L_i)\,,\\
\mathcal{A}_{\mathrm{U}(1)_i-\mathrm{SO}(2M)^2} &\sim \frac{1}{12}\int_{\mathcal{M}} N_i\, c_1(L_i)\, c_2(T) + \int_{\mathcal{M}} N_i\, \mathrm{ch}_3(L_i)\,.
\end{aligned} \tag{2.12}$$

The corresponding expressions for the $\mathrm{U}(1)_m$ factors with $m \in \{K+1, \dots, K+L\}$ are obtained by replacing everywhere $N_i\, \mathrm{ch}_k(L_i)$ by $\mathrm{ch}_k(V_m)$. This reflects the fact that in the latter case, we have internal $\mathrm{U}(N_m)$ bundles, while for $i \in \{1, \dots, K\}$, the $\mathrm{SU}(N_i)$ factor is external and appears only as an overall factor N_i in all Chern characters.

The field theory anomalies in the case where $\mathrm{SO}(2M)$ is part of the bundle have the same form (2.12) after the modified tadpole condition,

$$\widehat{\mathrm{Tad}} = c_2(T) + \sum_{i=1}^{K} N_i \mathrm{ch}_2(L_i) + \sum_{m=K+1}^{K+L} \mathrm{ch}_2(V_m) - \frac{1}{2} c_2(V_0) = 0\,, \tag{2.13}$$

is taken into account. The mixed anomalies $\mathcal{A}_{\mathrm{U}(1)_x-\mathrm{SO}(2M)^2}$ are of course absent in this case.

3. Green Schwarz mechanism for the heterotic $\mathrm{SO}(32)$ string in 4D

The anomalies encountered in the four-dimensional effective field theory have to be cancelled by a generalised Green-Schwarz (GS) mechanism for consistency of the models. As we show here, the couplings of the $h_{11}+1$ four-dimensional axions to the gauge fields and gravity arising from the ten-dimensional kinetic and one-loop counter term have precisely

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the correct form. In addition, we will explicitly derive the mass terms for the various axionic fields in order to determine which linear combinations of U(1)s remain massless in the low-energy effective field theory.

Let us first discuss the couplings which arise from the dimensional reduction of the ten-dimensional Green-Schwarz one-loop counter term [19] ($\ell_s \equiv 2\pi\sqrt{\alpha'}$)

$$S_{GS} = \frac{1}{48(2\pi)^3 \ell_s^2} \int B^{(2)} \wedge X_8 \tag{3.1}$$

with the eight-form given by

$$X_8 = \frac{1}{24}\,\mathrm{Tr}\,F^4 - \frac{1}{7200}\left(\mathrm{Tr}\,F^2\right)^2 - \frac{1}{240}\left(\mathrm{Tr}\,F^2\right)\left(\mathrm{tr}\,R^2\right) + \frac{1}{8}\,\mathrm{tr}\,R^4 + \frac{1}{32}\left(\mathrm{tr}\,R^2\right)^2 \tag{3.2}$$

for both the SO(32) and $E_8 \times E_8$ heterotic theories where Tr denotes the trace in the adjoint representation and tr the one in the fundamental. Although the eight-form is the same for SO(32) and $E_8 \times E_8$, the resulting four-dimensional effective couplings turn out to differ since in the latter case there is no independent fourth order Casimir.

In the SO(32) case, dimensional reduction of the GS counter term to four dimensions gives

$$\begin{aligned}
S_{GS} = & \frac{1}{(2\pi)^3 \ell_s^2} \int B \wedge \frac{1}{288}\,\mathrm{Tr}(F\overline{F}^3) - && (3.3)\\
& -\frac{1}{(2\pi)^3 \ell_s^2} \int B \wedge \frac{1}{5760}\,\mathrm{Tr}(F\overline{F}) \wedge \left(\frac{1}{15}\,\mathrm{Tr}\,\overline{F}^2 + \mathrm{tr}\,\overline{R}^2\right) + && (3.4)\\
& +\frac{1}{(2\pi)^3 \ell_s^2} \int B \wedge \left(\frac{1}{192}\,\mathrm{Tr}(F^2\overline{F}^2) - \frac{1}{86400}[\mathrm{Tr}(F\overline{F})]^2\right) - && (3.5)\\
& -\frac{1}{(2\pi)^3 \ell_s^2} \int B \wedge \frac{1}{11520}\,\mathrm{Tr}(F^2) \wedge \left(\frac{1}{15}\,\mathrm{Tr}\,\overline{F}^2 + \mathrm{tr}\,\overline{R}^2\right) + && (3.6)\\
& +\frac{1}{(2\pi)^3 \ell_s^2} \int B \wedge \frac{1}{768}\,\mathrm{tr}\,R^2 \wedge \left(\mathrm{tr}\,\overline{R}^2 - \frac{1}{15}\,\mathrm{Tr}\,\overline{F}^2\right), && (3.7)
\end{aligned}$$

where we denote by $\overline{F}$ gauge fields taking values along the compact directions and F along the four-dimensional non-compact ones. The two-form B has both legs chosen along the non-compact directions in the first two lines and along the compact directions in the remaining three terms. The expressions (3.3), (3.4) are mass terms for the U(1) gauge factors. (3.5) and (3.6) lead to vertex couplings of the axions with two gauge fields and finally the expression (3.7) gives rise to vertex couplings of the axions and two gravitons.

As in [10], there are additional mass terms and vertex couplings arising from the kinetic term in the ten-dimensional effective action

$$S_{\mathrm{kin}} = -\frac{1}{4\,\kappa_{10}^2} \int e^{-2\phi_{10}}\, H \wedge \star_{10} H\,, \tag{3.8}$$

where $\kappa_{10}^2 = \frac{1}{2}(2\pi)^7\,(\alpha')^4$ and the heterotic 3-form field strength $H = dB^{(2)} - \frac{\alpha'}{4}(\omega_Y - \omega_L)$ involves the gauge and gravitational Chern-Simons terms. In terms of the six-form $B^{(6)}$ dual to $B^{(2)}$, $\star_{10}\, dB^{(2)} = e^{2\phi_{10}}\, dB^{(6)}$, (3.8) contains

$$S_{\mathrm{kin}} = \frac{1}{8\pi \ell_s^6} \int \left(\mathrm{tr}\,F^2 - \mathrm{tr}\,R^2\right) \wedge B^{(6)}\,. \tag{3.9}$$

The discussion of the GS mechanism upon compactification necessitates the extraction of the various vertex couplings and mass terms from the above contributions to the four-dimensional lagrangian. It is convenient to introduce a basis ω_k, $(k = 1, \ldots, h_{11})$ of $H^2(\mathcal{M}, \mathbb{Z})$ together with the Hodge dual four-forms $\widehat{\omega}_k$ normalised such that $\int_{\mathcal{M}} \omega_k \wedge \widehat{\omega}_{k'} = \delta_{k,k'}$. In terms of these, the expansion of the various fields is

$$\begin{aligned} B^{(2)} &= b_0^{(2)} + \ell_s^2 \sum_{k=1}^{h_{11}} b_k^{(0)}\, \omega_k\,, & \operatorname{tr} \overline{F}^2 &= (2\pi)^2 \sum_{k=1}^{h_{11}} (\operatorname{tr} \overline{F}^2)_k\, \widehat{\omega}_k\,, \\ \overline{f}^m &= 2\pi \sum_{k=1}^{h_{11}} \overline{f}_k^m\, \omega_k\,, & \operatorname{tr} \overline{R}^2 &= (2\pi)^2 \sum_{k=1}^{h_{11}} (\operatorname{tr} \overline{R}^2)_k\, \widehat{\omega}_k\,, \\ B^{(6)} &= \ell_s^6\, b_0^{(0)} \operatorname{vol}_6 + \ell_s^4 \sum_{k=1}^{h_{11}} b_k^{(2)}\, \widehat{\omega}_k\,. \end{aligned} \tag{3.10}$$

Here vol_6 represents the volume form of the internal Calabi-Yau manifold normalised such that $\int_{\mathcal{M}} \operatorname{vol}_6 = 1$.

The traces occurring in the kinetic and counter terms have to be evaluated for a concrete choice of internal bundle. The results for the internal bundle as given in (2.2) are listed in appendix B, where we also summarise our notation used in the remainder of this paper. With these results at hand, it is a simple task to collect the explicit mass and GS terms.

From (3.3), (3.4) we find that the four-dimensional two-form field $b_0^{(2)}$ is rendered massive by the coupling to the abelian gauge fields given by

$$S^0_{\text{mass}} = \sum_{x=1}^{K+L} \frac{1}{6(2\pi)^5 \alpha'} \int_{\mathbb{R}_{1,3}} b_0^{(2)} \wedge f_x \int_{\mathcal{M}} \left(\operatorname{tr} \overline{F}_x^3 - \frac{1}{16} \operatorname{tr} \overline{F}_x \wedge \operatorname{tr} \overline{R}^2 \right), \tag{3.11}$$

where we denote by $\operatorname{tr}(\overline{F}_x^n)$ the trace in the fundamental representation of the $\mathrm{U}(N_x)$-bundle from which the abelian factor f_x descends. Again, in (3.11) and in all expressions which follow it is understood that the non-abelian part of $\overline{F}_x$ is present only for $x \in \{K+1, \ldots, K+L\}$. Traces over the fundamental representation of an $\mathrm{SU}(N_i)$ factor are denoted by $\operatorname{tr}_i(\widehat{F}^n)$ for distinction.

In addition, (3.9) gives rise to mass terms for the internal two-forms $b_k^{(2)}$,

$$S_{\text{mass}} = \sum_{x=1}^{K+L} \sum_{k=1}^{h_{11}} \frac{1}{(2\pi)^2 \alpha'} \int_{\mathbb{R}_{1,3}} (b_k^{(2)} \wedge f_x)(\operatorname{tr} \overline{F}_x)_k\,. \tag{3.12}$$

As for the gauge anomalies, the GS counter terms (3.5) and (3.6) provide the anomalous couplings of the axions to the external gauge fields,

$$S_{GS} = \frac{1}{2\pi} \int_{\mathbb{R}_{1,3}} \sum_{k=1}^{h_{11}} \sum_{i=1}^{K} b_k^{(0)} \operatorname{tr}_i \widehat{F}^2 \left(\frac{1}{4} (\overline{f}_i^2)_k - \frac{1}{192} (\operatorname{tr} \overline{R}^2)_k \right) - \tag{3.13}$$

$$- \frac{1}{384} \frac{1}{2\pi} \int_{\mathbb{R}_{1,3}} \sum_{k=1}^{h_{11}} b_k^{(0)} \operatorname{tr}_{\mathrm{SO}(2M)} F^2 \, (\operatorname{tr} \overline{R}^2)_k + \tag{3.14}$$

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$$+\frac{1}{2\pi}\int_{\mathbb{R}_{1,3}}\sum_{k=1}^{h_{11}}\sum_{x=1}^{K+L} b_k^{(0)}\, f_x^2\left(\frac{1}{4}\,(\mathrm{tr}\,\overline{F}^2{}_x)_k-\frac{N_x}{192}(\mathrm{tr}\,\overline{R}^2)_k\right). \tag{3.15}$$

Combining them with (3.12) and taking into account the analogous vertex couplings

$$S^0_{GS}=\frac{1}{8\pi}\int_{\mathbb{R}_{1,3}} b_0^{(0)}\left(2\sum_{i=1}^{K}\mathrm{tr}_i\,\widehat{F}^2+\mathrm{tr}_{\mathrm{SO}(2M)}\,F^2+2\sum_{x=1}^{K+L}N_x\,f_x^2\right) \tag{3.16}$$

from (3.9) together with its counterpart (3.11) leads to an expression for the respective anomaly six-form. For the mixed $\mathrm{U}(1)_x-\mathrm{SU}(N_i)$ anomaly, for instance, we find

$$\begin{aligned}\mathcal{A}_{\mathrm{U}(1)_x-\mathrm{SU}(N_i)^2}\sim\ &\frac{1}{12(2\pi)^5\alpha'}\,f_x\wedge\mathrm{tr}_i\,\widehat{F}^2\,\times\\ &\times\int_{\mathcal{M}}\left(\mathrm{tr}\,\overline{F}_x^3+3\ \mathrm{tr}\,\overline{F}_x\wedge\overline{f}_i^2-\frac{1}{8}\,\mathrm{tr}\,\overline{F}_x\wedge\mathrm{tr}\,\overline{R}^2\right),\end{aligned} \tag{3.17}$$

which is just tailor-made to cancel the mixed $\mathrm{U}(1)_x-\mathrm{SU}(N_i)^2$ anomaly. The cancellation pattern for the remaining abelian-non-abelian, cubic abelian and mixed gravitational anomalies follows the same lines. In the latter case the internal axion-graviton couplings follow from (3.7) and the external one from (3.9). Let us just list the resulting anomaly six-forms

$$\begin{aligned}\mathcal{A}_{\mathrm{U}(1)_x-\mathrm{SO}(2M)^2}&\sim\frac{1}{24(2\pi)^5\alpha'}\,f_x\wedge\mathrm{tr}_{\mathrm{SO}(2M)}\,F^2\int_{\mathcal{M}}\left(\mathrm{tr}\,\overline{F}_x^3-\frac{1}{8}\,\mathrm{tr}\,\overline{F}_x\wedge\mathrm{tr}\,\overline{R}^2\right),\\ \mathcal{A}_{\mathrm{U}(1)_x-G_{\mu\nu}^2}&\sim-\frac{1}{24(2\pi)^5\alpha'}f_x\wedge\mathrm{tr}\,R^2\int_{\mathcal{M}}\left(\mathrm{tr}\,\overline{F}_x^3-\frac{1}{16}\,\mathrm{tr}\,\overline{F}_x\wedge\mathrm{tr}\,\overline{R}^2\right),\\ \mathcal{A}_{\mathrm{U}(1)_x-\mathrm{U}(1)_y^2}&\sim\frac{1}{12(2\pi)^5\alpha'}f_x\wedge f_y^2\int_{\mathcal{M}}\left(N_y\,(\mathrm{tr}\,\overline{F}_x^3-\frac{1}{8}\,\mathrm{tr}\,\overline{F}_x\wedge\mathrm{tr}\,\overline{R}^2)+\mathrm{tr}\,\overline{F}_x\wedge\mathrm{tr}\,\overline{F}_y^2\right)\end{aligned} \tag{3.18}$$

and point out that they are in perfect agreement with the field theoretic anomalies given in the previous section. As usual, the anomalous U(1)s are rendered massive and therefore remain in the low-energy domain as perturbative global symmetries. The situation parallels that in type-I [21] and heterotic $E_8\times E_8$-theory [10], where the number of massive abelian factors is at least as large as that of the anomalous ones and in general given by the rank of the mass matrix

$$\mathrm{M}_{xk}=\begin{cases}\dfrac{1}{(2\pi)^2\alpha'}(\mathrm{tr}\,\overline{F}_x)_k & \text{for } k\in\{1,\ldots,h_{11}\}\\ \dfrac{1}{6(2\pi)^5\alpha'}\displaystyle\int_{\mathcal{M}}\left(\mathrm{tr}\,\overline{F}_x^3-\frac{1}{16}\,\mathrm{tr}\,\overline{F}_x\wedge\mathrm{tr}\,\overline{R}^2\right) & \text{for } k=0.\end{cases} \tag{3.19}$$

4. Non-universal gauge kinetic functions

Let us now derive the gauge kinetic functions [22, 23, 8, 24]. The holomorphic gauge kinetic function f_a appears in the four-dimensional effective field theory as

$$\mathcal{L}_{YM}=\frac{1}{4}\,\mathrm{Re}(\mathrm{f}_a)\,F_a\wedge\star F_a+\frac{1}{4}\ \mathrm{Im}(\mathrm{f}_a)\,F_a\wedge F_a\,. \tag{4.1}$$

With the definition of the complexified dilaton and Kähler moduli

$$S = \frac{1}{2\pi}\left[e^{-2\phi_{10}}\frac{\mathrm{Vol}(\mathcal{M})}{\ell_s^6} + i\, b_0^{(0)}\right], \qquad\qquad T_k = \frac{1}{2\pi}\left[-\alpha_k + i b_k^{(0)}\right], \tag{4.2}$$

the gauge kinetic functions can be read off from their imaginary parts in (3.13)–(3.16) to be

$$\begin{aligned}
\mathrm{f}_{\mathrm{SU}(N_i)} &= 2S + \sum_{k=1}^{h_{11}} T_k \left((\overline{f}_i^2)_k - \frac{1}{48}(\mathrm{tr}\,\overline{R}^2)_k \right), \\
\mathrm{f}_{\mathrm{SO}(2M)} &= S - \frac{1}{96}\sum_{k=1}^{h_{11}} T_k\,(\mathrm{tr}\,\overline{R}^2)_k\,, \\
\mathrm{f}_x &= 2N_x S + \sum_{k=1}^{h_{11}} T_k \left((\mathrm{tr}\,\overline{F}_x^2)_k - \frac{N_x}{48}(\mathrm{tr}\,\overline{R}^2)_k \right).
\end{aligned} \tag{4.3}$$

Note that the real parts of the gauge kinetic function are positive definite by definition. Therefore, requiring positivity of the expressions (4.3) in the perturbative regime, $g_s \ll 1$ and $r_i \gg 1$, imposes extra conditions on the allowed bundles. Away from the small coupling and large radii limit one expects both world-sheet and stringy instanton corrections to the gauge kinetic functions [8].

The real parts of the Kähler moduli are defined by the expansion of the Kähler form in terms of the chosen basis of two-cycles, $J = \ell_s^2 \sum_{i=1}^{h_{11}} \alpha_i \omega_i$, and the compact volume is computed from

$$\mathrm{Vol}(\mathcal{M}) = \frac{1}{6}\int_{\mathcal{M}} J\wedge J\wedge J = \frac{\ell_s^6}{6}\sum_{i,j,k} d_{ijk}\,\alpha_i\,\alpha_j\,\alpha_k\,, \tag{4.4}$$

where $d_{ijk} = \int_{\mathcal{M}} \omega_i\wedge\omega_j\wedge\omega_k$ are the triple intersection numbers of the basis of two-forms. The explicit computation of the real parts is more involved, however, the coupling to the dilaton can be directly seen from the dimensional reduction of the kinetic term of the gauge field,

$$S_{YM}^{(10)} = \frac{1}{2\kappa_{10}^2}\int e^{-2\phi_{10}}\,\frac{\alpha'}{4}\,\mathrm{tr}(F\wedge\star_{10}F)\,. \tag{4.5}$$

For $i \in \{1,\ldots,K\}$, the gauge kinetic function for the abelian factor of the $\mathrm{U}(N_i)$ group is proportional to the non-abelian part, $\mathrm{f}_{\mathrm{U}(1)_i} = N_i\,\mathrm{f}_{\mathrm{SU}(N_i)}$.

Note that in contrast to the $E_8\times E_8$ examples in [10], no off-diagonal couplings among abelian factors occur. Even more strikingly, the tree-level and one-loop corrected non-abelian and abelian gauge couplings of an observable $\mathrm{SU}(N_i)$ and $\mathrm{U}(1)_x$ gauge factor only depend on the internal gauge flux in the same $\mathrm{U}(N_i)$ and $\mathrm{U}(N_x)$ gauge group respectively. Since we used the same decomposition of SO(32) that naturally appears for intersecting D-branes, S-duality tells us that after all this result is not surprising. There, each stack of D-branes comes with its own gauge coupling determined by the size of the three-cycle the D6-branes are wrapping around.

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5. Fayet-Iliopoulos terms and supersymmetry conditions

The appearance of anomalous U(1) symmetries indicates the potential creation of Fayet-Iliopoulos (FI) terms in the four-dimensional effective action [25, 26, 6], which have to vanish for supersymmetry to be preserved. Note that in the following we ignore the option to cancel a non-vanishing FI-term by giving VEVs to charged scalars. In fact, we strongly believe that in more realistic scenarios which include fluxes, the induced mass terms for these fields will fix their VEVs at zero independently as is the case in the extensively studied type-IIB models [27]. From the general analysis of four-dimensional $\mathcal{N}=1$ supergravity it is well-known that the coefficients ξ_x of the FI-terms can be derived from the Kähler potential $\mathcal{K}$ via the relation

$$\frac{\xi_x}{g_x^2} = \left.\frac{\partial \mathcal{K}}{\partial V_x}\right|_{V=0} . \tag{5.1}$$

The gauge invariant Kähler potential relevant for our type of construction reads

$$\begin{aligned}\mathcal{K} = \frac{M_{pl}^2}{8\pi}\Bigg[&-\ln\Big(S+S^* - \sum_x Q_0^x\, V_x\Big) - \\ &-\ln\Big(- \sum_{i,j,k=1}^{h_{11}} \frac{d_{ijk}}{6}\Big(T_i + T_i^* - \sum_x Q_i^x\, V_x\Big)\Big(T_j + T_j^* - \sum_x Q_j^x\, V_x\Big) \times \\ &\times \Big(T_k + T_k^* - \sum_x Q_k^x\, V_x\Big)\Big)\Bigg]\end{aligned} \tag{5.2}$$

in the notation of [10]. The charges Q_k^x are defined via

$$S_{\text{mass}} = \sum_{x=1}^{K+L}\sum_{k=0}^{h_{11}} \frac{Q_k^x}{2\pi\alpha'} \int_{\mathbb{R}_{1,3}} f_x \wedge b_k^{(2)} \tag{5.3}$$

and can easily be read off from the mass terms (3.11) and (3.12).

The FI terms are seen to be proportional to the specific combination

$$\frac{\xi_x}{g_x^2} = -\frac{e^{2\phi_{10}} M_{pl}^2\, \ell_s^6}{4\mathrm{Vol}(\mathcal{M})}\left(\frac{1}{4} e^{-2\phi_{10}} \sum_{i,j,k=1}^{h_{11}} d_{ijk} Q_i^x \alpha_j \alpha_k - \frac{1}{2} Q_0^x\right) . \tag{5.4}$$

Inserting the concrete expressions for the charges immediately leads to the conclusion that the FI terms vanish if and only if

$$e^{-2\phi_{10}}\frac{1}{2}\int_{\mathcal{M}} J\wedge J\wedge \mathrm{tr}\overline{F}_x - \frac{(2\pi\alpha')^2}{3!}\int_{\mathcal{M}}\left(\mathrm{tr}\overline{F}_x^3 - \frac{1}{16}\,\mathrm{tr}\,\overline{F}_x \wedge \mathrm{tr}\,\overline{R}^2\right) = 0 \tag{5.5}$$

for each external $\mathrm{U}(1)_x$ factor separately. Note again that, as expected from the intersecting D-brane picture, the FI-term for $\mathrm{U}(1)_x$ only depends on the internal vector bundle with field strength $\overline{F}_x$. In contrast to the $E_8 \times E_8$ heterotic string, there appears the cubic term $\mathrm{tr}\overline{F}_x^3$ in the one-loop correction to the FI-term. This can be traced back to the fact that in contrast to E_8 the group SO(32) has an independent fourth order Casimir operator. It implies the well-known result that for the SO(32) heterotic string a bundle with structure group SU(N) generates a non-vanishing one-loop FI-term [5]. Again, away from the small string coupling and large radii limit one expects additional non-perturbative world-sheet and string instanton contributions to (5.5).

6. S-duality to the type-I string

So far we have derived all equations for the SO(32) heterotic string. We will now apply Heterotic-type-I S-duality to these equations and compare with known results on the type-I side.

6.1 The gauge couplings for type-I

First let us write the expression for the gauge couplings in a way which is more suitable for the S-duality transformation. The real part of the holomorphic gauge kinetic function f_x can be cast into the form

$$\mathrm{Re}(\mathrm{f}_x^H) = \frac{1}{\pi \ell_s^6} \left[\frac{N_x}{3!}\, g_s^{-2} \int_{\mathcal{M}} J \wedge J \wedge J - \frac{(2\pi\alpha')^2}{2} \int_{\mathcal{M}} J \wedge \left(\mathrm{tr}\, \overline{F}_x^2 - \frac{N_x}{48}\, \mathrm{tr}\, \overline{R}^2 \right) \right]. \tag{6.1}$$

Applying the heterotic-type-I string duality relations [28]

$$\begin{aligned} g_s^I &= (g_s^H)^{-1}\,, \\ J^I &= (g_s^H)^{-1} J^H \end{aligned} \tag{6.2}$$

with $g_s = e^{\phi_{10}}$ leads to

$$\mathrm{Re}(\mathrm{f}_x^I) = \frac{1}{\pi \ell_s^6 g_s} \left[\frac{N_x}{3!} \int_{\mathcal{M}} J \wedge J \wedge J - \frac{(2\pi\alpha')^2}{2} \int_{\mathcal{M}} J \wedge \left(\mathrm{tr}\, \overline{F}_x^2 - \frac{N_x}{48}\, \mathrm{tr}\, \overline{R}^2 \right) \right] \tag{6.3}$$

on the type-I side. Note that the second term has now become a perturbative α'-correction to the tree-level gauge coupling.

6.2 The non-abelian MMMS condition

The same S-duality relations (6.2) applied to the FI-terms (5.5) yield

$$\frac{1}{2} \int_{\mathcal{M}} J \wedge J \wedge \mathrm{tr} \overline{F}_x - \frac{(2\pi\alpha')^2}{3!} \int_{\mathcal{M}} \left(\mathrm{tr} \overline{F}_x^3 - \frac{1}{16}\, \mathrm{tr}\, \overline{F}_x \wedge \mathrm{tr}\, \overline{R}^2 \right) = 0 \tag{6.4}$$

on the type-I side, where the second term is again a perturbative α'-correction. We can combine the gauge kinetic function and the FI-term into a single complex quantity, the central charge

$$\mathcal{Z} = \int_{\mathcal{M}} \mathrm{tr}_{\mathrm{U}(N)} \left[\left(e^{J\, \mathrm{id} + i\mathcal{F}} \sqrt{\hat{A}(\mathcal{M})} \right) \right], \tag{6.5}$$

defined in terms of $\mathcal{F} = 2\pi\alpha' \overline{F}$ and the A-roof genus $\hat{A}(\mathcal{M}) = 1 + \frac{1}{48} \frac{1}{(2\pi)^2}\, \mathrm{tr}\, \mathcal{R}^2 + \cdots$ with $\mathcal{R} = \ell_s^2\, \overline{R}$. The gauge coupling and the FI-term are seen to be proportional to the real and imaginary part, respectively, of $\mathcal{Z}$.

In the case of abelian D9-branes in type-IIB we know that one can introduce an additional phase parameterising which $\mathcal{N}=1$ supersymmetry of the underlying $\mathcal{N}=2$ supersymmetry is preserved by the brane. Therefore, the general type-IIB supersymmetry condition is

$$\mathrm{Im}\left(\int_{\mathcal{M}} \mathrm{tr}_{\mathrm{U}(N)}\left[e^{i\varphi}\, e^{J\,\mathrm{id}+i\mathcal{F}}\,\sqrt{\hat{A}(\mathcal{M})}\right]\right)=0\,. \tag{6.6}$$

As usual in type-IIB theory coupled to a brane, we have now defined $\mathcal{F}=2\pi\alpha'\overline{F}+B\,\mathrm{id}$, thus taking into account the fact that for open strings only this combination is a gauge invariant quantity.

Note that (6.5) is precisely the perturbative part of the expression for the central charge as it appears in the Π-stability condition [18] for general B-type branes.[3] To our knowledge the form of this expression has never been derived from first principles. Rather, we understand that the central charge has been designed in such a way as to keep in analogy with the well-known RR-charge of the B-brane as seen in the Chern-Simons action — it is simply assumed that in the geometric limit, the two quantities coincide [29].

We find it quite interesting though not unexpected that, starting from the well-known Green-Schwarz anomaly terms, our four-dimensional effective field theory analysis leads precisely to the perturbative part of the Π-stability condition for B-type branes.

Equation (6.6) is also the integrability condition for the non-abelian generalisation of the MMMS equation for D9-branes in a curved background. The abelian version of this equation has been proven (without the curvature terms) in [30] starting from the DBI action of a single D-brane and it has been confirmed by a world-sheet calculation in [31]. Up to now it is strictly speaking only a conjecture that it can easily be generalised to (6.6) [32, 33]. However, our analysis relies exclusively on quantities of the four-dimensional $\mathcal{N}=1$ effective supergravity theory, the one-loop FI-term and the holomorphic gauge kinetic function. In particular, the non-renormalization theorems guarantee the absence of further perturbative corrections, thus dictating (6.6) as the perturbatively exact integrability condition at least for D9-branes. The absence of a stringy one-loop correction was shown in [34]. Of course, there will be additional non-perturbative corrections, which in the $g_s\to 0$ limit make out the complete Π-stability expression [18].

The integrability condition is not yet sufficient for supersymmetry preservation, but has to be supplemented by the correct stability condition. This will be the direct generalisation of μ-stability, which is the valid notion of stability only at leading order in α' and g_s. To investigate this point further, we have to know the local supersymmetry equation for non-abelian D9-branes underlying (6.6). At first sight, this seems to be beyond the scope of our supergravity analysis:

All we can say for sure starting from (6.6) is that the local SUSY condition for D9-branes has to be of the form

$$\left[\mathrm{Im}\left(e^{i\varphi}\, e^{J\,\mathrm{id}+i\mathcal{F}}\,\sqrt{\hat{A}(\mathcal{M})}\right)\right]_{\mathrm{top}}+d\alpha_5=0\,, \tag{6.7}$$

[3] This is true at least for space filling branes in case we consider also non-abelian fields. Of course our analysis has nothing to say about lower-dimensional non-abelian branes.

where α_5 is a globally defined 5-form so that $d\alpha_5$ is gauge covariant. At least for compactifications on genuine Calabi-Yau manifolds, where $dJ = 0$ and $dH = 0$, we cannot find any 5-form of this type which is also invariant under the axionic U(1) gauge symmetry $B \to B + d\chi$, $A \to A - \chi$ and does lead to a non-vanishing $d\alpha_5$.

Therefore, we would like to conclude that the possible correction $d\alpha_5$ is absent and that indeed the local supersymmetry condition is given by

$$\left[\mathrm{Im}\left(e^{i\varphi}\, e^{J\,\mathrm{id}+i\mathcal{F}}\,\sqrt{\hat{A}(\mathcal{M})}\right)\right]_{\mathrm{top}} = 0\,. \tag{6.8}$$

The notion of stability relevant for (6.8) has been analysed in [33] and been called π-stability (to stress that it is only the perturbative part of Π-stability). In particular, the authors have shown that (6.8) has a unique solution precisely if the bundle is stable with respect to the deformed slope

$$\pi(V) = \mathrm{Arg}\left(\int_{\mathcal{M}} \mathrm{tr}_{\mathrm{U}(N)}\left[e^{J\,\mathrm{id}+i\mathcal{F}}\,\sqrt{\hat{A}(\mathcal{M})}\right]\right), \tag{6.9}$$

i.e. the phase of the central charge. For supersymmetric configurations, we need to ensure that all objects are BPS with respect to the same supersymmetry algebra. This is guaranteed by the integrability condition (6.6) (with $\varphi = 0$ in our case).

We have phrased our discussion of stability on the type-I respectively type-IIB side. It is clear, though, that all concepts translate directly into heterotic language. In particular, it would be exciting to identify the S-dual non-perturbative effects which make out heterotic Π-stability. This would necessitate also a further study of instantons in the $E_8 \times E_8$-string, analysed perturbatively in [10].

To conclude this brief interlude, we would like to emphasise that in practical applications, it remains of course a very difficult task to explicitly prove stability of the bundles, as is the case already for μ-stability. Note, however, that a supersymmetric bundle still has vanishing π-slope, so that, as with μ-stability, $H^0(\mathcal{M}, V) = 0$ is a necessary condition for a U(N) bundle to be π-stable. Apart from this non-trivial hint towards stability, one may only be able to show that the integrability condition (6.6) is satisfied. Taking this pragmatic point of view, just as has been customary in most explicit constructions in the literature so far, it is now possible to construct orientifolds with magnetised D-branes for the case that the D9-branes support not only abelian bundles but also non-abelian ones. This opens up a whole plethora of new model building possibilities and we will give two simple though not realistic examples in section 7.

7. Examples

As we have stressed several times already, model building in type-I string theory has mainly focused on compactifications involving abelian bundles on manifolds with almost trivial tangent bundle, i.e. the torus or orbifolds thereof. This, however, is clearly only the tip of the iceberg of possible constructions. Let us give a very simple illustration of how the use of non-abelian bundles on general Calabi-Yau manifolds admits new solutions to

reps.	Cohomology	χ
$(\mathbf{1})_2$	$H^*(\mathcal{M}, \bigwedge^2 V)$	155
$(\mathbf{26})_1$	$H^*(\mathcal{M}, V)$	-80

Table 3: Massless spectrum of a $H = \mathrm{SO}(26) \times \mathrm{U}(1)$ model on the Quintic.

the string consistency conditions. We will formally work on the heterotic side of the story, but as emphasised, the picture is easily translated to type-I theory.

7.1 A model on the Quintic

Consider for simplicity the most basic Calabi-Yau manifold, the Quintic, with Hodge numbers $(h_{21}, h_{11}) = (101, 1)$, intersection form

$$I_3 = 5\,\eta^3 \tag{7.1}$$

and

$$c_2(T) = 10\,\eta^2\,. \tag{7.2}$$

Having only one Kähler modulus, the Quintic nicely illustrates the virtue of non-abelian bundles. Namely, in order to cancel the positive contribution of the tangent bundle to the tadpole equation (2.6), we inevitably have to consider bundles with non-abelian structure group, since line bundles contribute in this case with the same sign as the tangent bundle. Following the well-known construction of vector bundles via exact sequences as reviewed in [10], we therefore turn on the non-abelian bundle V defined by the exact sequence

$$0 \to V \to \mathcal{O}(1)^{\oplus 5}|_{\mathcal{M}} \to \mathcal{O}(3) \oplus \mathcal{O}(4)|_{\mathcal{M}} \to 0\,, \tag{7.3}$$

from which the Chern characters are computed to be

$$c_1(V) = -2\,\eta\,, \qquad \mathrm{ch}_2(V) = -10\,\eta^2\,, \qquad \mathrm{ch}_3(V) = -\frac{43}{3}\,\eta^3\,.$$

The resulting structure group is $G = \mathrm{U}(3)$ and the observable gauge group $H = \mathrm{SO}(26) \times \mathrm{U}(1)$ with the chiral spectrum displayed in table 3. The abelian factor is anomalous and merely survives as a global symmetry.

The one-loop corrected DUY equation (5.5) leads to the freezing

$$r = \sqrt{\frac{91}{6}}\, g_s\,, \tag{7.4}$$

where the Kähler class has been expanded as $J = \ell_s^2\, r\, \eta$. For $g_s = 0.7$ one gets $r = 2.7$, so that it is possible to freeze a combination of the string coupling and the radius such that both remain in the perturbative regime.

The one-loop corrected gauge couplings are given by

$$\begin{aligned} \left.\frac{1}{g^2_{\mathrm{SO}(26)}}\right|_{1-\mathrm{loop}} &= \frac{95}{18\pi} r \sim 4.5\,, \\ \left.\frac{1}{g^2_{\mathrm{U}(1)}}\right|_{1-\mathrm{loop}} &= \frac{245}{3\pi} r \sim 70 \end{aligned}$$

for the above value of the string coupling.

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reps.	Cohomology	χ
$(\mathbf{16})_{1,0}$	$H^*(\mathcal{M},V_1)$	-24
$(\mathbf{16})_{0,1}$	$H^*(\mathcal{M},V_2)$	-24
$(\mathbf{1})_{1,1}$	$H^*(\mathcal{M},V_1\otimes V_2)$	-120
$(\mathbf{1})_{1,-1}$	$H^*(\mathcal{M},V_1\otimes V_2^*)$	0

Table 4: Massless spectrum of a $H = \mathrm{SO}(16)\times \mathrm{U}(1)\times \mathrm{U}(1)_{massive}$ model on a CICY with $h_{11}=2$.

7.2 An SO(16) model on a two-parameter CICY

The second example involving supersymmetric non-abelian bundles is defined on the CICY

$$\mathcal{M} = \begin{matrix}\mathbb{P}_3\\ \mathbb{P}_1\end{matrix}\begin{bmatrix}4\\2\end{bmatrix} \tag{7.5}$$

with Hodge numbers $(h_{21},h_{11})=(86,2)$, for which the intersection form

$$I_3 = 2\eta_1^3 + 4\,\eta_1^2\eta_2 \tag{7.6}$$

and the second Chern class of the tangent bundle

$$c_2(T) = 6\eta_1^2 + 8\eta_1\eta_2 \tag{7.7}$$

were computed e.g. in [10] in terms of the Kähler forms η_1,η_2 of the ambient product space. This time, we turn on two U(4) bundles V_i $(i=1,2)$ described both by the exact sequence

$$0\to V_i \to \mathcal{O}(1,0)^{\oplus 2}|_{\mathcal{M}} \oplus \mathcal{O}(0,1)^{\oplus 4}|_{\mathcal{M}} \to \mathcal{O}(2,1)^{\oplus 2}|_{\mathcal{M}} \to 0\,. \tag{7.8}$$

The topological data specifying V_i are therefore

$$c_1(V_i) = -2\eta_1 + 2\eta_2\,,\qquad \mathrm{ch}_2(V_i) = -3\eta_1^2 - 4\eta_1\eta_2\,,\qquad \mathrm{ch}_3(V_i) = -\frac{7}{3}\eta_1^3 - 3\eta_1^2\eta_2\,. \tag{7.9}$$

The sum of these two bundles satisfies the tadpole condition, and the observable non-abelian gauge group is SO(16). The linear combination $\mathrm{U}(1)_1 - \mathrm{U}(1)_2$ is anomaly-free while its orthogonal abelian factor is anomalous. The chiral spectrum is given in table 4.

The supersymmetry conditions for both bundles are of course identical and lead to

$$r_1\,(4r_2 - r_1) = \frac{67}{6} g_s^2 \tag{7.10}$$

in terms of our usual expansion of the Kähler from as $J=\ell_s^2\sum_{i=1}^2 r_i\eta_i$. The gauge couplings are given by

$$\begin{aligned}\left.\frac{1}{g^2_{\mathrm{SO}(16)}}\right|_{1-\mathrm{loop}} &= \frac{1}{\pi}\,\frac{\frac{50}{18}r_1^2 + 8r_1r_2 - 2r_2^2}{4r_2 - r_1}\,,\\ \left.\frac{1}{g^2_{\mathrm{U}(1)_i}}\right|_{1-\mathrm{loop}} &= \frac{1}{\pi}\,\frac{\frac{2}{9}r_1^2 + 140r_2^2 + 32r_2^2}{4r_2 - r_1}\,,\qquad i=1,2\end{aligned} \tag{7.11}$$

where we have used (7.10) to eliminate the string coupling.

We have demonstrated the possibility of reducing the rank of the observable gauge group, in particular of the unwanted $\mathrm{SO}(2M)$ remnant, with the help of non-abelian bundles. Of course more realistic models would have to involve in addition line bundles which would give rise to $\mathrm{SU}(N_i)$ factors as discussed. The search for appealing models of this kind is beyond the scope of this publication and remains for future investigation.

8. Phenomenological aspects

In this section we briefly address some of the qualitative phenomenological aspects of these heterotic SO(32) compactifications with non-abelian and abelian bundles. Some of them have already been mentioned in the sections before, but we nevertheless list them here for completeness. When choosing just abelian bundles, the phenomenological features are very similar to the intersecting D-brane models in type-I string theories. Please keep in mind that the methods developed so far are valid in the small coupling, large radius regime and that away from the perturbative limit certain low-energy parameters also receive world-sheet and stringy instanton corrections.

The size of the string scale. The phenomenological consequences of these heterotic string models drastically depend on the value of the string scale. It is known that in type-I models with non-spacetime filling D-branes the string scale can in principle be anywhere between the TeV range and the Planck scale. For the heterotic string the tree-level gravitational and gauge couplings are of the order

$$M_{pl}^2 \sim \frac{M_s^8\, V_6}{g_s^2}\,, \qquad \frac{1}{g_{YM}^2} \sim \frac{M_s^6\, V_6}{g_s^2}\,, \tag{8.1}$$

which immediately implies $M_{pl}^2 \sim M_s^2/g_{YM}^2$. For gauge couplings of order one at the unification scale, the string scale is therefore of the order of the Planck scale.

Let us now analyse the possible consequences of the higher string loop corrections. It is known that the Einstein-Hilbert term does not receive any stringy loop corrections [35], whereas for the gauge coupling the exact one-loop corrected expression is of the form

$$\frac{1}{g_{YM}^2} \sim \frac{M_s^6\, V_6}{g_s^2} - M_s^2\, V_2\, \beta\,. \tag{8.2}$$

Here V_2 denotes the two-cycle volume (transversal to the four-form flux) and β can be expressed by the various second Chern-characters appearing in (2.5) and is essentially an integer of order $10-10^2$ for most known examples. Note for more than one Kähler modulus, the one-loop correction actually contains a sum over all two-cycle volumes. For $\beta > 0$ (as is the case for the SO(16) gauge coupling in example 7.2) and assuming the gauge coupling to be of order one, we get

$$\frac{M_{pl}^2 - M_s^2}{M_s^2} \sim M_s^2\, V_2\, \beta\,. \tag{8.3}$$

This tells us that M_s can be significantly smaller than the Planck scale as long as the 2-cycle volume V_2 is large in units of M_s. To be more precise, we have to make sure that the lightest Kaluza-Klein mode of the Standard Model gauge sector is not lighter than the weak scale M_w. Thus, identifying $V_2 = 1/M_w^2$ we can lower the string scale down to

$$M_s \sim \frac{\sqrt{M_{pl}\, M_w}}{\beta^{\frac{1}{4}}} \simeq \frac{10^{11}}{\beta^{\frac{1}{4}}}\,\mathrm{GeV}\,. \tag{8.4}$$

The first equation in (8.1) implies that the "longitudinal" volume V_4, defined as $V_6 = V_2\, V_4$, is of order β in string units and therefore large enough β still allows V_4 in the perturbative regime. For $\beta < 0$ both the tree-level and the one-loop correction in (8.2) are positive and we arrive at the usual conclusion that $M_s \simeq M_{pl}$.

To summarize, provided that in a heterotic string compactification we can engineer the Standard Model gauge group via bundles with positive combinations of the second Chern classes β, the one-loop corrections allow us to lower the string scale down to the intermediate regime if the background is non-isotropic and has large (transversal) two-cycles. Whether this scenario can actually be realized in Standard-like models remains to be seen but it again shows that we ought to carefully reevaluate our prejudice on heterotic string compactifications.

The observable gauge group. Clearly, the observable gauge group H is the commutant of the structure group G in SO(32). For our first choice of the structure group this gauge group was $H = \mathrm{SO}(2M) \times \prod_{j=1}^{K} \mathrm{SU}(N_j)$ extended by the anomaly free part of the $\mathrm{U}(1)^{K+L}$ gauge factors. Therefore the effect of truly non-abelian $\mathrm{U}(N_m)$ factors in the structure group is to reduce the rank of the initial gauge group SO(32).

In general there are multiple anomalous U(1) gauge factors, which become massive via some bilinear couplings to the Kähler-axions and the universal axion. The scale of each mass term is of the order of the string scale; however for multiple anomalous U(1)s we can arrange for mass eigenvalues in the entire phenomenologically acceptable range from the weak scale up to the string scale. Therefore, one might consider the existence of additional massive U(1) gauge factors as the main model independent prediction of this kind of string compactifications. For Standard-like models one has to ensure that there exists a $\mathrm{U}(1)_Y$ which remains massless after the Green-Schwarz mechanism has been taken into account. Note that for the $E_8 \times E_8$ heterotic string there exist choices of the structure group which do not lead to any observable U(1).

The matter spectrum. The chiral matter comes in various bifundamental and (anti-)symmetric representations of the observable gauge group, and the net number of generations is given by the Euler characteristic of the tensor product of the bundles in question (see table 1). For the general embedding we are considering there never occurs any spinor representation of the $\mathrm{SO}(2M)$ part of the observable gauge group. Thus, for realising SO(10) GUTs one has to consider compactificatons of the $E_8 \times E_8$ heterotic string.

If one wishes to determine the complete chiral and non-chiral spectrum, knowledge of the index is of course not enough. One really has to compute the various cohomology classes. Here also additional massless states can appear in the singlet and adjoint representations of the gauge group. These are additional potential moduli fields and correspond in the S-dual type-I formulation to open string moduli. By working at a generic point in moduli space we expect that the amount of non-chiral matter is reduced [36, 37] as compared to the mostly studied special points in moduli space like orbifolds, free-fermion constructions and Gepner models.

The gauge couplings. We have derived the perturbative expressions for the non-abelian and abelian gauge couplings. Besides the — up to normalisation — universal tree-level result there appears a one-loop threshold correction. Including these corrections, both the non-abelian and the abelian gauge couplings depend on the Kähler moduli and are non-universal, which is also expected from the S-dual type-I picture. Quite analogously also, the gauge kinetic functions of the anomaly-free abelian factors are given by the corresponding linear combinations of the gauge kinetic functions of the original U(1) factors. For concrete Standard-like models taking into account the light charged matter fields, one has to check case by case whether these moduli dependent gauge couplings can give rise to gauge coupling unification at the string scale.

Supersymmetry. For a specific observable gauge group and charged matter content, i.e. without giving VEVs to charged scalars, the D-terms are determined by the Fayet-Iliopoulos terms. As we have shown these contain a tree-level and a stringy one-loop correction and do depend on the Kähler moduli. Therefore, for $\mathrm{U}(N)$ bundles supersymmetry implies that certain combinations of the dilaton and the Kähler moduli get fixed. For pure $\mathrm{SU}(N)$ bundles with non-vanishing third Chern class, however, the tree level FI-term vanishes and one gets only a moduli independent one-loop induced FI-term. It follows that in this case supersymmetric vacua can at best be reached by turning on VEVs for charged scalars.

We have no real control over the F-terms, where world-sheet instantons can generate non-vanishing contributions. It is known that for $(0,2)$ models which admit an anomaly-free linear sigma model description no world-sheet instantons destabilize the background [38–40]. However, this is not necessarily true for the general tadpole cancelling configurations of abelian and non-abelian gauge bundles we consider here, where the left-moving U(1) on the world-sheet is anomalous. It would be interesting to systematically study these globally anomalous linear sigma models.

Fluxes and moduli stabilisation. For the heterotic string one can also turn on a background H_3 form flux. This induces a superpotential of the form [41–43]

$$W_{\text{flux}} \sim \int \Omega \wedge (H + i\, dJ) \tag{8.5}$$

and allows to stabilise the complex structure moduli. Here we have also included the torsion piece, which of course vanishes in the supergravity limit. In contrast to the type-IIB case, the dilaton does not appear in the superpotential and therefore cannot be stabilised in this way. As a result, stabilisation of the Kähler, dilaton and bundle moduli seems only possible by taking into account non-perturbative contributions to the superpotential. [44–46].

Yukawa couplings. For phenomenological purposes it is very important to compute the Yukawa couplings for this class of models. The physical Yukawa couplings can only be read

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off if the kinetic terms are canonically normalised, which means that the physical Yukawa couplings involve both the Kähler potential and the superpotential. The Kähler potential for charged fields might be hard to derive, but the superpotential should already allow us to find some selection rules for the non-vanishing couplings. At large radius these selection rules are expected to be given by the cohomology ring $H^q(\mathcal{M},\mathcal{W})$, i.e.

$$H^p(\mathcal{M},\mathcal{W}_1)\times H^q(\mathcal{M},\mathcal{W}_2)\times H^r(\mathcal{M},\mathcal{W}_3)\to H^3(\mathcal{M},\mathcal{O})=\mathbb{C}\,,$$

where the bundles $\mathcal{W}_i$ are to be taken from table 1. For anomaly-free linear sigma models these cohomology rings could be computed as chiral (topological) rings of the underlying conformal field theory [47−49]. It remains to be investigated how these couplings can be explicitly found for more general bundles corresponding to anomalous linear sigma models.

Soft supersymmetry breaking terms. Soft supersymmetry breaking in the observable gauge sector can either be achieved by some breaking in the hidden sector of a given model, e.g. via gaugino condensation, or through suitable three-form fluxes. Once one knows the gauge kinetic function, the Kähler potential and the superpotential, the induced soft-terms can be computed by the general supergravity formulas [50−52], which are parameterised by the supersymmetry breaking auxiliary fields F_s in the chiral superfields.

Stability of the proton. As in the intersecting D-brane scenario, for a concrete realization of the Standard Model baryon and lepton number might appear as gauged anomalous U(1) symmetries [21] which become massive via the Green-Schwarz mechanism and therefore survive as perturbative global symmetries. Clearly in such models there would be no problem with rapid proton decay.

9. Conclusions

The aim of this article is to generalise the construction of string compactifications on smooth Calabi-Yau spaces with non-abelian and in particular abelian vector bundles to the case of the SO(32) heterotic string. We worked out the generalised Green-Schwarz mechanism in very much detail and computed the one-loop threshold corrected gauge kinetic functions and the FI-terms in the perturbative regime. In this sort of topological sector, we found completely S-dual features on the heterotic and type-I sides. The only difference, dictated by S-duality, is that certain α' corrections in type-I theory are mapped to string loop corrections on the heterotic side.

We conclude that for smooth heterotic/type-I compactifications all features are in accordance with S-duality and that the seeming differences can be traced back to the restricted set of examples one has considered so far and probably to some subtleties related to the fixed points of orbifold models. Having discussed the smooth case in detail, it still remains to be understood how heterotic orbifolds and free-fermion constructions precisely fit into our approach.

As a byproduct of our effective four dimensional analysis, we were able to confirm from first principles, i.e. from the celebrated Green-Schwarz terms, the perturbative part of the Π-stability condition for B-type branes.

So far we have only laid the foundation and qualitatively discussed a couple of phenomenological aspects of this class of SO(32) heterotic compactifications. A more systematic study of all the string vacua with abelian and non-abelian bundles has to follow. More examples and the question of whether Standard-like models can be constructed this way remain for future investigation. This might include a study of the statistics of the vacua of this type. It would also be interesting to investigate for concrete models the possibility of moduli stabilisation via fluxes [53, 54].

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A. Anomalies in terms of Euler characteristics

The cubic non-abelian, mixed and pure abelian anomalies can be expressed in terms of the Euler characteristics as follows for the case that the structure group is $G = \prod_{m=K+1}^{K+L} \mathrm{SU}(N_m) \times \mathrm{U}(1)^{K+L}$,

$$\begin{aligned}
\mathcal{A}_{\mathrm{SU}(N_i)^3} &\sim A_{\mathrm{SU}(N_i)^3} = (N_i-4)\chi(L_i^2) + \sum_{j\neq i} N_j \left(\chi(L_i\otimes L_j) + \chi(L_i\otimes L_j^{-1})\right) + \\
&\qquad + \sum_m \left(\chi(V_m\otimes L_i) + \chi(V_m^*\otimes L_i)\right) + 2M\chi(L_i)\,,\\
\mathcal{A}_{\mathrm{U}(1)_i-\mathrm{SU}(N_i)^2} &\sim N_i\chi(L_i^2) + A_{\mathrm{SU}(N_i)^3}\,,\\
\mathcal{A}_{\mathrm{U}(1)_i-\mathrm{SU}(N_{j\neq i})^2} &\sim A_{\mathrm{U}(1)_i-\mathrm{SU}(N_{j\neq i})^2} = N_i\left(\chi(L_i\otimes L_j) + \chi(L_i\otimes L_j^{-1})\right),\\
\mathcal{A}_{\mathrm{U}(1)_m-\mathrm{SU}(N_i)^2} &\sim A_{\mathrm{U}(1)_m-\mathrm{SU}(N_i)^2} = \chi(V_m\otimes L_i) - \chi(V_m^*\otimes L_i)\,,\\
\mathcal{A}_{\mathrm{U}(1)_i-G_{\mu\nu}^2} &\sim 3N_i\chi(L_i^2) + A_{\mathrm{SU}(N_i)^3}\,,\\
\mathcal{A}_{\mathrm{U}(1)_m-G_{\mu\nu}^2} &\sim A_{\mathrm{U}(1)_m-G_{\mu\nu}^2} = 2\chi(\textstyle\bigwedge^2 V_m) + \displaystyle\sum_i N_i\left(\chi(V_m\otimes L_i) - \chi(V_m^*\otimes L_i)\right) + \\
&\qquad + \sum_{n\neq m}\left(\chi(V_m\otimes V_n) + \chi(V_m\otimes V_n^*)\right) + 2M\chi(V_m)\,,\\
\mathcal{A}_{\mathrm{U}(1)_i^3} &\sim 3N_i^2\chi(L_i^2) + N_i A_{\mathrm{SU}(N_i)^3}\,,\\
\mathcal{A}_{\mathrm{U}(1)_i-\mathrm{U}(1)_{j\neq i}^2} &\sim N_j A_{\mathrm{U}(1)_i-\mathrm{SU}(N_{j\neq i})^2}\,,\\
\mathcal{A}_{\mathrm{U}(1)_i-\mathrm{U}(1)_m^2} &\sim N_i\left(\chi(V_m\otimes L_i) + \chi(V_m^*\otimes L_i)\right),\\
\mathcal{A}_{\mathrm{U}(1)_m-\mathrm{U}(1)_i^2} &\sim N_i A_{\mathrm{U}(1)_m-\mathrm{SU}(N_i)^2}\,,\\
\mathcal{A}_{\mathrm{U}(1)_m^3} &\sim 6\chi(\textstyle\bigwedge^2 V_m) + A_{\mathrm{U}(1)_m-G_{\mu\nu}^2}\,,\\
\mathcal{A}_{\mathrm{U}(1)_m-\mathrm{U}(1)_n^2} &\sim \chi(V_m\otimes V_n) + \chi(V_m\otimes V_n^*)\,,\\
\mathcal{A}_{\mathrm{U}(1)_i-\mathrm{SO}(2M)^2} &\sim N_i\chi(L_i)\,,\\
\mathcal{A}_{\mathrm{U}(1)_m-\mathrm{SO}(2M)^2} &\sim \chi(V_m)\,.
\end{aligned}$$

If $\mathrm{SO}(2M)$ also belongs to the bundle, the following modifications occur: in $A_{\mathrm{SU}(N_i)^3}$ we make the replacement $2M\chi(L_i) \to \chi(V_0 \otimes L_i)$; in $A_{\mathrm{U}(1)_m - G^2_{\mu\nu}}$ we substitute $2M\chi(V_m) \to \chi(V_0 \otimes V_m)$.

B. Collection of trace identities

The detailed computation of the GS counter terms depends on the concrete embedding of the structure group into $\mathrm{SO}(32)$. In this appendix we list some useful trace identities which enter the analysis of the GS mechanism in section 3. We restrict ourselves here to the choice of bundles made in (2.2), but completely analogous expressions arise for all other embeddings.

To shorten the notation, we treat the internal background $\mathrm{U}(N_m)$-bundles V_m and the line bundles L_i on the same footing and denote them simply by V_x, $x \in \{1, \dots, K+L\}$. It turns out to be convenient to decompose the corresponding field strength $\overline{F}_x$ into its non-abelian $\mathrm{SU}(N_x)$-part $\widehat{\overline{F}}_x$ (where present) and the diagonal $\mathrm{U}(1)$-part $\overline{f}_x$ as

$$\overline{F}_x = \widehat{\overline{F}}_x + \overline{f}_x \, \mathrm{I}_{N_x \times N_x} \,, \tag{B.1}$$

with $\mathrm{I}_{N_x \times N_x}$ denoting the identity matrix. In other words, it is understood that the non-abelian part of $\overline{F}_x$ is simply absent for $x \in \{1, \dots, K\}$, where the background bundles are merely line bundles. However, even though in this case the $\mathrm{SU}(N_x)$ is part of the external gauge theory, we insist on the presence of the identity matrix as a remnant to compactly keep track of various factors of N_x.

Let us note explicitly that (B.1) implies the obvious identities

$$\begin{aligned}
\mathrm{tr}\, \overline{F}_x &= N_x \overline{f}_x \,, \\
\mathrm{tr}\, \overline{F}_x^2 &= \mathrm{tr}\, \widehat{\overline{F}}{}_x^2 + N_x (\overline{f}_x)^2 \,, \\
\mathrm{tr}\, \overline{F}_x^3 &= \mathrm{tr}\, \widehat{\overline{F}}{}_x^3 + 3\overline{f}_x \; \mathrm{tr}\, \widehat{\overline{F}}{}_x^2 + N_x (\overline{f}_x)^3 \,,
\end{aligned} \tag{B.2}$$

where all traces are in the fundamental representation.

Analogously, unbarred quantities denote the respective four-dimensional gauge field strengths.

With these conventions, it is a simple exercise to decompose the traces in the adjoint of $\mathrm{SO}(32)$, Tr, of the various combinations of external and internal fields appearing in the GS counter terms into traces over the fundamental of the $\mathrm{U}(N_x)$-factors,

$$\begin{aligned}
\mathrm{Tr}\, F\overline{F}^3 &= 12 \sum_{x=1}^{K+L} f_x \wedge \left(4 \;\, \mathrm{tr}\, \overline{F}_x^3 + \mathrm{tr}\, \overline{F}_x \sum_{y=1}^{K+L} \mathrm{tr}\, \overline{F}_y^2 \right) , \\
\mathrm{Tr}\, F^2 \overline{F}^2 &= 4 \sum_{i=1}^{K} \mathrm{tr}_i\, \widehat{F}^2 \wedge \left(12 \; (\overline{f}^i)^2 + \sum_{x=1}^{K+L} \mathrm{tr}\, \overline{F}_x^2 \right) + \\
&\quad + 4 \sum_{x=1}^{K+L} (f_x)^2 \wedge \left(12 \;\, \mathrm{tr}\, \overline{F}_x^2 + N_x \sum_{y=1}^{K+L} \mathrm{tr}\, \overline{F}_y^2 \right) +
\end{aligned}$$

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$$
\begin{aligned}
& +8\sum_{x,y=1}^{K+L} f_x\, f_y \wedge \mathrm{tr}\,\overline{F}_x\ \mathrm{tr}\,\overline{F}_y +\ 2\ \mathrm{tr}_{\mathrm{SO}(2M)}\, F^2 \wedge \sum_{x=1}^{K+L} \mathrm{tr}\,\overline{F}_x^2\,, \\
\mathrm{Tr}\, F^2 &= 30\ \mathrm{tr}_{\mathrm{SO}(2M)}\, F^2 + 60 \sum_{x=1}^{K+L} \mathrm{tr}\, F_x^2\,, \\
\mathrm{Tr}\, F\overline{F} &= 60 \sum_{x=1}^{K+L} f_x \wedge \mathrm{tr}\,\overline{F}_x\,, \\
\mathrm{Tr}\,\overline{F}^2 &= 60 \sum_{x=1}^{K+L} \mathrm{tr}\,\overline{F}_x^2\,.
\end{aligned} \tag{B.3}
$$

It follows immediately that the GS counter terms computed in section 3 have precisely the right form to cancel the field theoretic anomalies.

References

[1] R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, *Toward realistic intersecting D-brane models*, hep-th/0502005.

[2] L.E. Ibáñez, R. Rabadán and A.M. Uranga, *Anomalous U(1)'s in type-I and type-IIB $D = 4$, $N = 1$ string vacua*, *Nucl. Phys.* **B 542** (1999) 112 [hep-th/9808139].

[3] Z. Lalak, S. Lavignac and H.P. Nilles, *String dualities in the presence of anomalous U(1) symmetries*, *Nucl. Phys.* **B 559** (1999) 48 [hep-th/9903160].

[4] A. Sagnotti, *A note on the Green-Schwarz mechanism in open string theories*, *Phys. Lett.* **B 294** (1992) 196 [hep-th/9210127].

[5] M. Dine, N. Seiberg and E. Witten, *Fayet-Iliopoulos terms in string theory*, *Nucl. Phys.* **B 289** (1987) 589.

[6] J.J. Atick, L.J. Dixon and A. Sen, *String calculation of Fayet-Iliopoulos d terms in arbitrary supersymmetric compactifications*, *Nucl. Phys.* **B 292** (1987) 109.

[7] D. Cremades, L.E. Ibáñez and F. Marchesano, *SUSY quivers, intersecting branes and the modest hierarchy problem*, *JHEP* **07** (2002) 009 [hep-th/0201205].

[8] H.P. Nilles and S. Stieberger, *String unification, universal one-loop corrections and strongly coupled heterotic string theory*, *Nucl. Phys.* **B 499** (1997) 3 [hep-th/9702110].

[9] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *Noncommutative compactifications of type-I strings on tori with magnetic background flux*, *JHEP* **10** (2000) 006 [hep-th/0007024].

[10] R. Blumenhagen, G. Honecker and T. Weigand, *Loop-corrected compactifications of the heterotic string with line bundles*, *JHEP* **06** (2005) 020 [hep-th/0504232].

[11] M.B. Green, J.H. Schwarz and P.C. West, *Anomaly free chiral theories in six-dimensions*, *Nucl. Phys.* **B 254** (1985) 327.

[12] A. Strominger, *Superstrings with torsion*, *Nucl. Phys.* **B 274** (1986) 253.

[13] J. Distler and B.R. Greene, *Aspects of (2,0) string compactifications*, *Nucl. Phys.* **B 304** (1988) 1.

[14] G. Aldazabal, A. Font, L.E. Ibáñez and A.M. Uranga, *New branches of string compactifications and their F-theory duals*, *Nucl. Phys.* **B 492** (1997) 119 [hep-th/9607121].

[15] P. Berglund, A. Klemm, P. Mayr and S. Theisen, *On type-IIB vacua with varying coupling constant*, *Nucl. Phys.* **B 558** (1999) 178 [hep-th/9805189].

[16] B. Andreas and D. Hernandez Ruiperez, *U(n) vector bundles on Calabi-Yau threefolds for string theory compactifications*, hep-th/0410170.

[17] A. Lukas and K.S. Stelle, *Heterotic anomaly cancellation in five dimensions*, *JHEP* **01** (2000) 010 [hep-th/9911156].

[18] M.R. Douglas, B. Fiol and C. Romelsberger, *Stability and BPS branes*, hep-th/0002037.

[19] M.B. Green and J.H. Schwarz, *Anomaly cancellation in supersymmetric $D = 10$ gauge theory and superstring theory*, *Phys. Lett.* **B 149** (1984) 117.

[20] E. Witten, *Some properties of O(32) superstrings*, *Phys. Lett.* **B 149** (1984) 351.

[21] L.E. Ibáñez, F. Marchesano and R. Rabadán, *Getting just the standard model at intersecting branes*, *JHEP* **11** (2001) 002 [hep-th/0105155].

[22] J.P. Derendinger, L.E. Ibáñez and H.P. Nilles, *On the low-energy limit of superstring theories*, *Nucl. Phys.* **B 267** (1986) 365.

[23] L.E. Ibáñez and H.P. Nilles, *Low-energy remnants of superstring anomaly cancellation terms*, *Phys. Lett.* **B 169** (1986) 354.

[24] S. Stieberger, *(0,2) heterotic gauge couplings and their M-theory origin*, *Nucl. Phys.* **B 541** (1999) 109 [hep-th/9807124].

[25] M. Dine, N. Seiberg, X.G. Wen and E. Witten, *Nonperturbative effects on the string world sheet*, *Nucl. Phys.* **B 278** (1986) 769.

[26] M. Dine, N. Seiberg, X.G. Wen and E. Witten, *Nonperturbative effects on the string world sheet, 2*, *Nucl. Phys.* **B 289** (1987) 319.

[27] M.P. Garcia del Moral, *A new mechanism of kahler moduli stabilization in type-IIB theory*, hep-th/0506116.

[28] J. Polchinski and E. Witten, *Evidence for heterotic — type-I string duality*, *Nucl. Phys.* **B 460** (1996) 525 [hep-th/9510169].

[29] P.S. Aspinwall, *D-branes on Calabi-Yau manifolds*, hep-th/0403166.

[30] M. Marino, R. Minasian, G.W. Moore and A. Strominger, *Nonlinear instantons from supersymmetric p-branes*, *JHEP* **01** (2000) 005 [hep-th/9911206].

[31] A. Kapustin and Y. Li, *Stability conditions for topological D-branes: a worldsheet approach*, hep-th/0311101.

[32] R. Minasian and A. Tomasiello, *Variations on stability*, *Nucl. Phys.* **B 631** (2002) 43 [hep-th/0104041].

[33] H. Enger and C.A. Lutken, *Non-linear Yang-Mills instantons from strings are pi-stable D-branes*, *Nucl. Phys.* **B 695** (2004) 73 [hep-th/0312254].

[34] E. Poppitz, *On the one loop fayet-iliopoulos term in chiral four dimensional type-I orbifolds*, *Nucl. Phys.* **B 542** (1999) 31 [hep-th/9810010].

[35] I. Antoniadis, E. Gava and K.S. Narain, *Moduli corrections to gravitational couplings from string loops*, *Phys. Lett.* **B 283** (1992) 209 [hep-th/9203071].

[36] V. Braun, Y.-H. He, B.A. Ovrut and T. Pantev, *A standard model from the* $E(8) \times E(8)$ *heterotic superstring*, *JHEP* **06** (2005) 039 [hep-th/0502155].

[37] V. Braun, Y.-H. He, B.A. Ovrut and T. Pantev, *A heterotic standard model*, *Phys. Lett.* **B 618** (2005) 252 [hep-th/0501070].

[38] E. Silverstein and E. Witten, *Criteria for conformal invariance of (0,2) models*, *Nucl. Phys.* **B 444** (1995) 161 [hep-th/9503212].

[39] A. Basu and S. Sethi, *World-sheet stability of (0,2) linear sigma models*, *Phys. Rev.* **D 68** (2003) 025003 [hep-th/0303066].

[40] C. Beasley and E. Witten, *Residues and world-sheet instantons*, *JHEP* **10** (2003) 065 [hep-th/0304115].

[41] K. Becker, M. Becker, K. Dasgupta and P.S. Green, *Compactifications of heterotic theory on non-Kähler complex manifolds, i*, *JHEP* **04** (2003) 007 [hep-th/0301161].

[42] K. Becker, M. Becker, K. Dasgupta and S. Prokushkin, *Properties of heterotic vacua from superpotentials*, *Nucl. Phys.* **B 666** (2003) 144 [hep-th/0304001].

[43] G.L. Cardoso, G. Curio, G. Dall'Agata and D. Lüst, *BPS action and superpotential for heterotic string compactifications with fluxes*, *JHEP* **10** (2003) 004 [hep-th/0306088].

[44] G. Curio and A. Krause, *G-fluxes and non-perturbative stabilisation of heterotic M-theory*, *Nucl. Phys.* **B 643** (2002) 131 [hep-th/0108220].

[45] M. Becker, G. Curio and A. Krause, *de Sitter vacua from heterotic M-theory*, *Nucl. Phys.* **B 693** (2004) 223 [hep-th/0403027].

[46] G. Curio, A. Krause and D. Lüst, *Moduli stabilization in the heterotic/iib discretuum*, hep-th/0502168.

[47] R. Blumenhagen, R. Schimmrigk and A. Wisskirchen, *The (0,2) exactly solvable structure of chiral rings, Landau-Ginzburg theories and Calabi-Yau manifolds*, *Nucl. Phys.* **B 461** (1996) 460 [hep-th/9510055].

[48] A. Adams, J. Distler and M. Ernebjerg, *Topological heterotic rings*, hep-th/0506263.

[49] S. Katz and E. Sharpe, *Notes on certain (0,2) correlation functions*, hep-th/0406226.

[50] L.E. Ibáñez and D. Lüst, *Duality anomaly cancellation, minimal string unification and the effective low-energy lagrangian of 4-D strings*, *Nucl. Phys.* **B 382** (1992) 305 [hep-th/9202046].

[51] V.S. Kaplunovsky and J. Louis, *Model independent analysis of soft terms in effective supergravity and in string theory*, *Phys. Lett.* **B 306** (1993) 269 [hep-th/9303040].

[52] A. Brignole, L.E. Ibáñez and C. Muñoz, *Towards a theory of soft terms for the supersymmetric standard model*, *Nucl. Phys.* **B 422** (1994) 125 [hep-ph/9308271].

[53] K. Becker and K. Dasgupta, *Heterotic strings with torsion*, *JHEP* **11** (2002) 006 [hep-th/0209077].

[54] G.L. Cardoso et al., *Non-kaehler string backgrounds and their five torsion classes*, *Nucl. Phys.* **B 652** (2003) 5 [hep-th/0211118].